

MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2018

15-04-2018 Online (Morning)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of **Physics, Mathematics and Chemistry** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. Candidates will be awarded marks as started above in instruction No. 5 for correct response of each question. $\frac{1}{4}$ (one fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from that total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars / marking responses on Side-1 and Side-2 of the Answer Sheet use only Black Ball Point Pen provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in four pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is D. Make sure that the CODE printed on Side-2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet

PART-A-PHYSICS

1. A solution containing active cobalt ${}^{60}_{27}\text{Co}$ having activity of $0.8 \mu\text{Ci}$ and decay constant λ is injected in an animal's body. If 1cm^3 of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body? ($1 \text{ Ci} = 3.7 \times 10^{10}$ decays per second and at $t=10 \text{ hrs } e^{-\lambda t} = 0.84$)
- (1) 7 liters (2) 4 liters (3) 6 liters (4) 5 liters

Ans. 4

Sol. Let total volume of blood is v , initial activity $A_0 = 0.8 \mu\text{Ci}$ its activity at time $t = A = A_0 e^{-\lambda t}$ activity of x

$$\text{volume } A^1 = \left(\frac{A}{V}\right)x = x\left(\frac{A_0}{V}\right)e^{-\lambda t}$$

$$V = x\left(\frac{A_0}{A^1}\right)e^{-\lambda t}$$

$$V = (1\text{cm}^3) \left(\frac{8 \times 10^{-7} \times 3.7 \times 10^{10}}{\frac{300}{60}} \right) (0.84)$$

$$= 4.97 \times 10^3 \text{ cm}^3 = 4.97 \text{ litre}$$

2. A body of mass m is moving in a circular orbit of radius R about a planet of mass M . At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is :

- (1) $-\frac{GMm}{6R}$ (2) $+\frac{GMm}{6R}$ (3) $-\frac{GMm}{2R}$ (4) $\frac{GMm}{2R}$

Ans. 1

Sol. $E_i = -\frac{GMm}{2R}$

$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R} = -\frac{4GMm}{6R} = -\frac{2Mm}{3R}$$

$$E_f - E_i = \frac{GMm}{R} \left(-\frac{2}{3} + \frac{1}{2} \right) = -\frac{GMm}{6R}$$

3. A given object takes n times more time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is :

(1) $\sqrt{\frac{1}{1-n^2}}$

(2) $\sqrt{1-\frac{1}{n^2}}$

(3) $1-\frac{1}{n^2}$

(4) $\frac{1}{2-n^2}$

Ans. 3

Sol. Time taken to slide along smooth surface

$$s = \frac{1}{2}g\sin 45^\circ t_1^2$$

$$t_1 = \sqrt{\frac{2\sqrt{2}s}{g}}$$

Time taken to slide along rough surface

$$S = \frac{1}{2}(g\sin 45^\circ - \mu g\cos 45^\circ)t_2^2$$

$$t_2 = \sqrt{\frac{2\sqrt{2}s}{g(1-\mu)}}$$

$$t_2 = nt_1$$

$$\frac{2\sqrt{2}s}{g(1-\mu)} = n^2 \times \frac{2\sqrt{2}s}{g} \Rightarrow 1-\mu = \frac{1}{n^2} \Rightarrow \mu = 1-\frac{1}{n^2}$$

4. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding) :

(1) 150 m

(2) 75 m

(3) 100 m

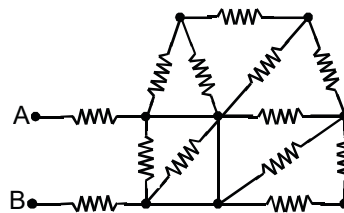
(4) 160 m

Ans. 4

Sol. $S = \frac{u^2}{2a}$

$$\frac{S_1}{S_2} = \frac{u_1^2}{u_2^2} \Rightarrow S_2 = \left(\frac{u_2}{u_1}\right)^2 S_1 = (2)^2(40) = 160m$$

5. In the given circuit all resistances are of value R ohm each. The equivalent resistance between A and B is:



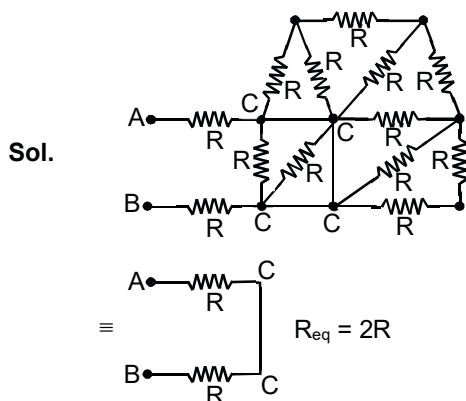
(1) $\frac{5R}{3}$

(2) $\frac{5R}{2}$

(3) 2R

(4) 3R

Ans. 3



6. An ideal capacitor of capacitance $0.2 \mu\text{F}$ is charged to a potential difference of 10 V . The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self inductance 0.5 mH . The current at a time when the potential difference across the capacitor is 5 V , is :

- (1) 0.17 A (2) 0.34 A (3) 0.25 A (4) 0.15 A

Ans. 1

Sol. Using energy conservation

$$\frac{1}{2} \times 0.2 \times 10^{-6} \times 10^2 + 0 = \frac{1}{2} \times 0.2 \times 10^{-6} \times 5^2 + \frac{1}{2} \times 0.5 \times 10^{-3} I^2$$

$$I = \sqrt{3} \times 10^{-1} \text{ A} = 0.17 \text{ A}$$

7. A body of mass M and charge q is connected to a spring of spring constant k . It is oscillating along x -direction about its equilibrium position, taken to be at $x = 0$, with an amplitude A . An electric field E is applied along the x -direction. Which of the following statements is correct?

(1) The new equilibrium position is at a distance $\frac{qE}{2k}$ from $x = 0$.

(2) The total energy of the system is $\frac{1}{2} m\omega^2 A^2 - \frac{1}{2} \frac{q^2 E^2}{k}$

(3) The total energy of the system is $\frac{1}{2} m\omega^2 A^2 + \frac{1}{2} \frac{q^2 E^2}{k}$

(4) The new equilibrium position is at a distance $\frac{2qE}{k}$ from $x = 0$

Ans. 3

Sol. Equilibrium position will shift to point where resultant force = 0

$$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$$

$$\frac{1}{2} m\omega^2 \left[A^2 + \left(\frac{qE}{k} \right)^2 \right] = \frac{1}{2} m\omega^2 A^2 + \frac{1}{2} \frac{q^2 E^2}{k}$$

8. The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is :

(1) 20 eV (2) 34 eV (3) 79 eV (4) 109 eV

Ans. 3

Sol. Energy required to remove e⁻ from singly ionized helium atom = 54.4 eV

Energy required to remove e⁻ form helium atom = x eV given 54.4 eV = 2.2x ⇒ x = 24.73 eV

Energy required to ionize helium atom = 79.12 eV

9. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is :

(1) 0.6 (2) 10^{-2} (3) 6 (4) 2

Ans. 4

Sol. $F_1 = \frac{GM_e m}{r_1^2}$ $F_2 = \frac{GM_e M_s}{r_2^2}$

$\Delta F_1 = -\frac{2GM_e m}{r_1^3} \Delta r_1$ $\Delta F_2 = -\frac{2GM_e M_s}{r_2^3} \Delta r_2$

$\frac{\Delta F_1}{\Delta F_2} = \frac{m \Delta r_1}{r_1^3} \frac{r_2^3}{M_s \Delta r_2} = \left(\frac{m}{M_s}\right) \left(\frac{r_2^3}{r_1^3}\right) \left(\frac{\Delta r_1}{\Delta r_2}\right)$

using $\Delta r_1 = \Delta r_2 = 2 R_{\text{earth}}$

$m = 8 \times 10^{22}$ kg

$M_s = 2 \times 10^{30}$ kg

$r_1 = 0.4 \times 10^6$ km

$r_2 = 150 \times 10^6$ km

we get $\frac{\Delta F_1}{\Delta F_2} = 2$

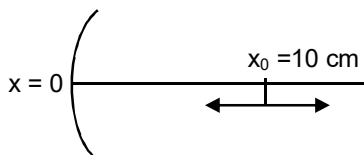
10. In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is :

(1) 0.2150 cm (2) 0.3150 cm (3) 0.0430 cm (4) 0.4300 cm

Ans. 1

Sol. Least Count = $\frac{0.25}{5 \times 100}$ cm = 5×10^{-4} cm
 Reading = 4×0.05 cm + $30 \times 5 \times 10^{-4}$ cm
 = $(0.2 + 0.0150)$ cm = 0.2150 cm

11. A particle is oscillating on the X-axis with an amplitude 2 cm about the point. $X_0 = 10$ cm, with a frequency ω . A concave mirror of focal length 5 cm is placed at the origin (see figure).



Identify the correct statements.

- (A) The image executes periodic motion.
- (B) The image executes non-periodic motion.
- (C) The turning points of the image are asymmetric w.r.t. the image of the point at $x = 10$ cm.
- (D) The distance between the turning points of the oscillation of the image is $\frac{100}{21}$ cm.

- (1) (B), (C) (2) (A), (D) (3) (B), (D) (4) (A) (C) (D)

Ans. 4

Sol. When object is at 8 cm

$$V_1 = \frac{f \times u}{u - f} = -\frac{40}{3} \text{ cm}$$

When object is at 12 cm

$$V_2 = -\frac{60}{7} \text{ cm}$$

$$\text{Separation} = |V_1 - V_2| = \frac{100}{21} \text{ cm}$$

So, A, C and D are correct

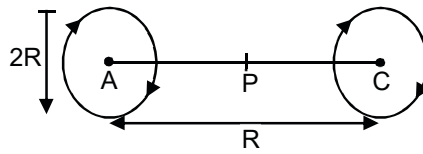
12. One mole of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, 27°C. The work done on the gas will be :

- (1) 300 R ln 2 (2) 300 R ln 6 (3) 300 R ln 7 (4) 300 R

Ans. 1

Sol. Work done on gas = $nRT \ln\left(\frac{p_f}{p_i}\right) = R(300) \ln(2) = 300R \ln 2$

13. A Helmholtz coil has a pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by [Refer to figure given below] :

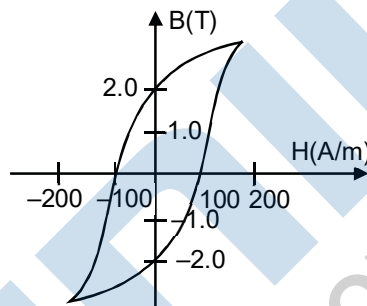


- (1) $\frac{8N\mu_0 I}{5^{3/2}R}$ (2) $\frac{4N\mu_0 I}{5^{1/2}R}$ (3) $\frac{8N\mu_0 I}{5^{1/2}R}$ (4) $\frac{4N\mu_0 I}{5^{3/2}R}$

Ans. 1

Sol.
$$B = 2 \left(\frac{\mu_0 N I R^2}{2 \left(R^2 + \frac{R^2}{4} \right)^{3/2}} \right) = \frac{\mu_0 N I R^2}{\frac{5^{3/2}}{8}} = \frac{8\mu_0 N I}{5^{3/2}R}$$

14. The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solenoid with 1000 turns/cm. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is :



- (1) 20 μ A (2) 40 μ A (3) 2 mA (4) 1 mA

Ans. 4

Sol. Coercivity of Ferro magnet $H = 100$ A/m

$nI = 100$

$I = \frac{100}{10^5} = 1\text{mA}$

15. A Carnot's engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is:

- (1) 2100 J (2) 2520 J (3) 420 J (4) 772 J

Ans. 3

Sol. Efficiency = $1 - \frac{T_2}{T_1} = \frac{W}{Q_2 + W}$

$\Rightarrow 1 - \frac{250}{300} = \frac{W}{Q_2 + W}$

$W = \frac{Q_2}{5} = \frac{500 \times 4.2}{5} \text{J} = 420\text{J}$

16. A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe? (Speed of sound in air is 340 ms^{-1})
 (1) 220 cm (2) 190 cm (3) 200 cm (4) 180 cm

Ans. 3

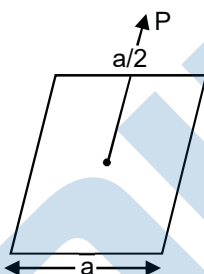
Sol. Organ pipe will have frequency either 255 or 257 Hz

Using 255Hz

$$255 = \frac{3V}{2l} \qquad \ell = \frac{3 \times 340}{2 \times 255} \text{ m}$$

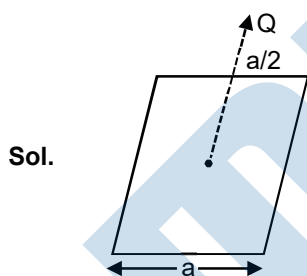
$$\ell = 200 \text{ cm}$$

17. A charge Q is placed at a distance $a/2$ above the centre of the square surface of edge a as shown in the figure



- (1) $\frac{Q}{6\epsilon_0}$ (2) $\frac{Q}{2\epsilon_0}$ (3) $\frac{Q}{3\epsilon_0}$ (4) $\frac{Q}{\epsilon_0}$

Ans. 1

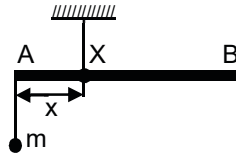


Sol.

charged particle can be Considered at centre of a cube of side a, and given surface represents its one side.

So flux $\phi = \frac{Q}{6\epsilon_0}$

18. A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variables that give a straight line, when plotted, are:



- (1) $m, \frac{1}{x^2}$ (2) $m, \frac{1}{x}$ (3) m, x^2 (4) m, x

Ans. 2

Sol.

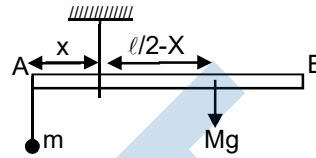
Balancing torque w.r.t. point of suspension

$$mgx = Mg\left(\frac{l}{2} - x\right)$$

$$mx = M\frac{l}{2} - Mx$$

$$m = \left(M\frac{l}{2}\right)\frac{1}{x} - M$$

$$y = \alpha\frac{1}{x} - C \quad \text{equation of a straight line}$$



19. A monochromatic beam of light has a frequency $\nu = \frac{3}{2\pi} \times 10^{12}$ Hz and is propagating along the direction

$\frac{\hat{i} + \hat{j}}{\sqrt{2}}$. It is polarized along the k direction. The acceptable form for the magnetic field is :

- (1) $\frac{E_0}{C} \hat{k} \cos \left[10^4 \frac{(\hat{i} + \hat{j})}{\sqrt{2}} \cdot \vec{r} + (3 \times 10^{12})t \right]$ (2) $\frac{E_0}{C} \frac{(\hat{i} - \hat{j})}{\sqrt{2}} \cos \left[10^4 \frac{(\hat{i} + \hat{j})}{\sqrt{2}} \cdot \vec{r} - (3 \times 10^{12})t \right]$
 (3) $\frac{E_0}{C} \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} \cos \left[10^4 \frac{(\hat{i} + \hat{j})}{\sqrt{2}} \cdot \vec{r} + (3 \times 10^{12})t \right]$ (4) $\frac{E_0}{C} \frac{(\hat{i} - \hat{j})}{\sqrt{2}} \cos \left[10^4 \frac{(\hat{i} + \hat{j})}{\sqrt{2}} \cdot \vec{r} + (3 \times 10^{12})t \right]$

Ans. 2

Sol. $\hat{E} \times \hat{B}$ should give direction of wave propagation

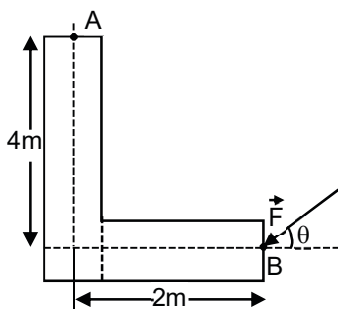
$$\Rightarrow \hat{K} \times \hat{B} \parallel \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

option - 1 $\hat{K} \times \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right) = \frac{\hat{j} - (-\hat{i})}{\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} \parallel \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

option -2 and 4 does not satisfy this.

wave propagation vector \hat{K} should be along $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

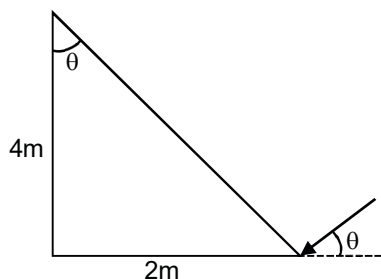
20. A force of 40N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by :



- (1) $\tan \theta = \frac{1}{2}$ (2) $\tan \theta = 4$ (3) $\tan \theta = 2$ (4) $\tan \theta = \frac{1}{4}$

Ans. 1

Sol. Moment of force will be maximum when line of action of force is perpendicular to line AB.



$$\tan \theta = \frac{2}{4} = \frac{1}{2}$$

21. The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz width for the highest modulating frequency 15 kHz will be :

- (1) 10 (2) 15 (3) 8 (4) 20

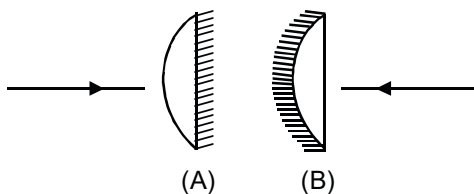
Ans. 4

Sol. If modulating frequency is 15 KHz then band width of one channel = 30 kHz

$$\text{No of channels accommodate} = \frac{300\text{kHz}}{30\text{kHz}} = 10$$

22. A planoconvex lens becomes an optical system of 28 cm focal length when its plane surface is silvered and illuminated from left to right as shown in Fig-A.

If the same lens is instead silvered on the curved surface and illuminated from other side as in Fig. B, it acts like an optical system of focal length 10 cm. The refractive index of the material of lens is :



- (1) 1.50 (2) 1.55 (3) 1.75 (4) 1.51

Ans. 2

Sol. Case-1

$$\frac{1}{f_1} = \left(\frac{\mu-1}{R}\right) \quad f = -28$$

$$P = 2P_1 + P_2$$

$$\frac{1}{28} = 2\left(\frac{\mu-1}{R}\right)$$

Case-2

$$\frac{1}{f_1} = \left(\frac{\mu-1}{R}\right) \quad f_2 = -\frac{R}{2} \quad f = -10\text{cm}$$

$$P = 2P_1 + P_2 \Rightarrow \frac{1}{10} = 2\left(\frac{\mu-1}{2}\right) + \frac{2}{R}$$

$$\frac{1}{10} = \frac{1}{28} + \frac{2}{R}$$

$$\frac{2}{R} = \frac{1}{10} - \frac{2}{28} = \frac{18}{280}$$

$$R = \frac{280}{9} \text{ cm}$$

$$\frac{1}{28} = 2\left(\frac{\mu-1}{\frac{280}{9}}\right)$$

$$\mu - 1 = \frac{5}{9}$$

$$\mu = 1 + \frac{5}{9} = \frac{14}{9} = 1.55$$

23. In a meter bridge, as shown in the figure, it is given that resistance $Y = 12.5\Omega$ and that the balance is obtained at a distance 39.5 cm from end A (by jockey J). After interchanging the resistance X and Y, a new balance point is found at a distance l_2 from end A. What are the values of X and l_2 ?

Ans. No option (correct answer : $\tan^{-1} \left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \right)$ closest option (3)

26. Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2 , their de Broglie wavelength in the frame of reference attached to their centre of mass is :

(1) $\lambda_{\text{CM}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$ (2) $\lambda_{\text{CM}} = \lambda_1 = \lambda_2$ (3) $\lambda_{\text{CM}} = \left(\frac{\lambda_1 + \lambda_2}{2} \right)$ (4) $\frac{1}{\lambda_{\text{CM}}} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

Ans. 1

Sol. Momentum of each electron $\frac{h}{\lambda_1} \hat{i}$ & $\frac{h}{\lambda_2} \hat{j}$

Velocity of centre of mass

$$V_{\text{cm}} = \frac{h}{2m\lambda_1} \hat{i} + \frac{h}{2m\lambda_2} \hat{j}$$

Velocity of 1st particle about centre of mass

$$V_{1\text{cm}} = \frac{h}{2m\lambda_1} \hat{i} - \frac{h}{2m\lambda_2} \hat{j}$$

$$\lambda_{\text{cm}} = \frac{h}{\sqrt{\frac{h^2}{4\lambda_1^2} + \frac{h^2}{4\lambda_2^2}}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

27. The relative error in the determination of the surface area of a sphere is α . Then the relative error in the determination of its volume is :

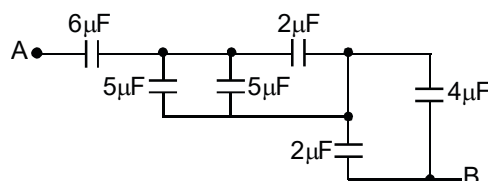
(1) $\frac{2}{3} \alpha$ (2) α (3) $\frac{5}{2} \alpha$ (4) $\frac{3}{2} \alpha$

Ans. 4

Sol. $\frac{\Delta s}{s} = 2 \times \frac{\Delta r}{r}$ $\frac{\Delta v}{v} = 3 \times \frac{\Delta r}{r}$

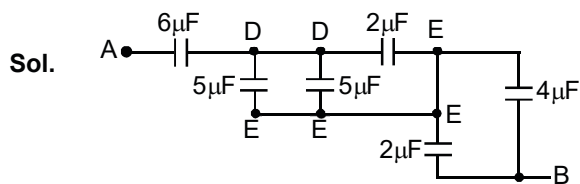
$$\frac{\Delta v}{v} = \frac{3}{2} \alpha$$

28. The equivalent capacitance between A and B in the circuit given below, is :

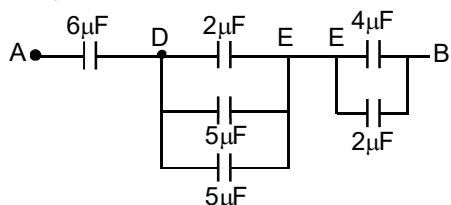


(1) 5.4 μF (2) 3.6 μF (3) 2.4 μF (4) 4.9 μF

Ans. 3



Simplified circuit



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12} \Rightarrow C_{eq} = \frac{12}{5} = 2.4 \mu F$$

29. In a common emitter configuration with suitable bias, it is given that R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by :

β is current gain, I_B , I_C and I_E are respectively base, collector and emitter currents.

(1) $\beta^2 \frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_E}, \beta^2 \frac{R_L}{R_{BE}}$

(2) $\beta \frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta^2 \frac{R_L}{R_{BE}}$

(3) $\beta^2 \frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta \frac{R_L}{R_{BE}}$

(4) $\beta \frac{R_L}{R_{BE}}, \frac{\Delta I_E}{\Delta I_B}, \beta^2 \frac{R_L}{R_{BE}}$

Ans. 2

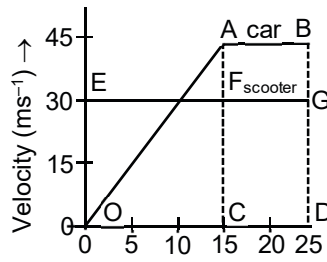
Sol. From NCERT

Current gain $\beta = \frac{\Delta I_C}{\Delta I_B}$

Voltage gain $A_v = \frac{\Delta V_{CE}}{R_{BE} \Delta I_B} = \beta \frac{R_L}{R_{BE}}$

Power gain $A_p = \beta A_v = \beta^2 \frac{R_L}{R_{BE}}$

30. The velocity-time graphs of a car and a scooter are shown in the figure. (i) The difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively.



- (1) 112.5 m and 22.5 s (2) 222.5 m and 10 s (3) 337.5 m and 25 s (4) 112.5 m and 15 s

Ans. 1

Sol. Distance travelled by car in 15 sec = $\frac{1}{2}(45)(15) = \frac{675}{2}$ m, Distance traveled by scooter in 15 seconds = $30 \times 15 = 450$

Let car catches scooter in time t;

$$\frac{675}{2} + 45(t - 15) = 30t$$

$$337.5 + 45t - 675 = 30t \Rightarrow 15t = 337.5 \Rightarrow t = 22.5 \text{ sec}$$

PART-B-MATHEMATICS

31. If $\lambda \in \mathbb{R}$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is

- (1) 20 (2) $2\sqrt{7}$ (3) $4\sqrt{2}$ (4*) $2\sqrt{5}$

Sol. $\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$$\alpha^3 + \beta^3 = (\lambda - 2)[(\lambda - 2)^2 - 3(10 - \lambda)]$$

Let $f(\lambda) = (\lambda - 2)^3 - 3(10 - \lambda)(\lambda - 2)$

$$\Rightarrow f'(\lambda) = 3(\lambda - 2)^2 + 3(\lambda - 2) - 3(10 - \lambda) = 0$$

$$\lambda = 4, -2$$

$$\Rightarrow f''(\lambda) = 6\lambda - 6 = 6(\lambda - 1)$$

Now $f''(4) > 0$ So, $f(\lambda)$ min at $\lambda = 4$

So, $\alpha - \beta = \pm \sqrt{20i} \Rightarrow |\alpha - \beta| = 2\sqrt{5}$.

32. An aeroplane flying at a constant speed, parallel to the horizontal ground, km above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30° , then the speed (in km/hr) of the aeroplane, is

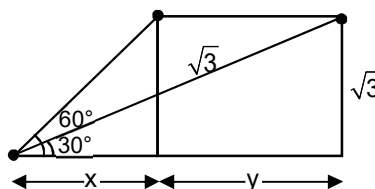
- (1*) 1440 (2) 720 (3) 1500 (4) 750

Sol. Let plane speed = K km/hr

So, $y = \frac{5}{3600} \times K \text{ km}$

Now $\tan 60^\circ = \frac{\sqrt{3}}{x} \Rightarrow x = 1 \text{ km}$

$\tan 30^\circ = \frac{\sqrt{3}}{x+y} \Rightarrow \frac{\sqrt{3}}{1 + \frac{5k}{3600}} = \frac{1}{\sqrt{3}} \Rightarrow K = 1440 \text{ km/hr}$



33. If x_1, x_2, \dots, x_n and $\frac{1}{h_1}, \frac{1}{h_2}, \dots, \frac{1}{h_n}$ are two A.P.s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 \cdot h_{10}$ equals

- (1) 1600 (2) 3200 (3*) 2560 (4) 2650

Sol. Let $x_1 = a$ and $c \cdot d = d$

and $\frac{1}{h_1} = A$ and $c \cdot 8 = D$

Now $x_8 = a + 7d = 20$ (1)

and $x_8 = a + 2d = 8$ (2)

by (1) and (2) $\Rightarrow a = \frac{16}{5}$ and $d = \frac{12}{5}$

So, $x_5 = \frac{64}{5}$ (3)

Now $\frac{1}{h_2} = \frac{1}{8} = A + D$ (4)

and $\frac{1}{h_7} = A + 6D = \frac{1}{20}$ (5)

by equation (4) and (5)

$A = \frac{28}{200}$ and $D = \frac{-3}{200}$

So, $\frac{1}{h_{10}} = \frac{1}{200} \Rightarrow h_{10} = 200$ (6)

by equation (3) and (6)

$x_5 \cdot h_{10} = \frac{64}{5} \times 200 = 2560$

34. A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is

(1) $\frac{x}{3} + \frac{y}{2} + \frac{z}{1} = 1$ (2) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{6}$ (3) $x + y + z = 6$ (4*) $\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$

Sol. Let A(a, 0, 0), B(0, b, 0) and C(0, 0, c)

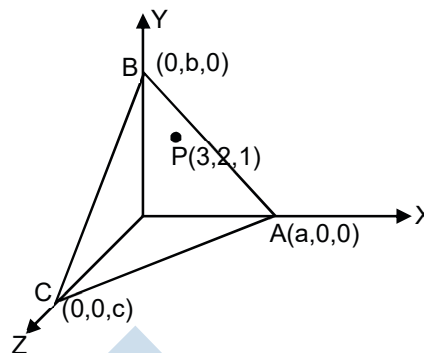
Plane ABC equation = $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

P(3, 2, 1) is on plane

So, $\frac{3}{a} + \frac{2}{b} + \frac{1}{c} = 1$

Now (a, b, c) are the point of intersection of all given planes so locus of intersection point is

$\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$



35. If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then the truth values of p, q and r are, respectively

- (1) T, T, T (2) F, F, F (3*) T, F, T (4) F, T, F

Sol.

p	q	r	$\sim p$	$\sim q$	$(p \wedge \sim q)$	$p \wedge r$	$(p \wedge \sim q) \wedge (p \wedge r)$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$
T	T	T	F	F	F	T	F	T	T
T	T	F	F	F	F	F	F	T	T
T	F	T	F	T	T	T	T	F	F
T	F	F	F	T	T	F	F	F	T
F	T	T	T	F	F	F	F	T	T
F	T	F	T	F	F	F	F	T	T
F	F	T	T	T	F	F	F	T	T
F	F	F	T	T	F	F	F	T	T

36. If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval

- (1) $(-\infty, -10]$ (2) $[10, \infty)$ (3) $(-10, 0)$ (4*) $(0, 10)$

Sol. $\frac{b}{1-r} = 5 \Rightarrow b = 5 - 5r$

We know that $-1 < r < 1$

So, $b \in (0, 10)$

37. n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is

- (1) 9 (2) 8 (3*) 7 (4) 6

Sol. $\boxed{\cdot \cdot \cdot \cdot \cdot \cdot} = 3^6 = 27 \times 27 = 729$

6 digit

$$\boxed{\cdot \cdot \cdot \cdot \cdot \cdot \cdot} = 3^7 = 2187$$

7 digit

38. If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$, then

$$\lim_{x \rightarrow 0} \frac{f'(x)}{x}$$

(1*) exists and is equal to -2

(2) does not exist

(3) exists and is equal to 2

(4) exists and is equal to 0

Sol.
$$\lim_{x \rightarrow 0} \frac{f'(x)}{x} = \lim_{x \rightarrow 0} \begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x & 2 \\ \tan x & x & 1 \end{vmatrix} + \lim_{x \rightarrow 0} \frac{1}{x} \begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix} + \lim_{x \rightarrow 0} \frac{1}{x} \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_3$$

$$R_2 \rightarrow R_2 - 2R_3$$

$$\Rightarrow \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} + \lim_{x \rightarrow 0} \frac{1}{x} \begin{vmatrix} \cos x - \tan x & 0 & 0 \\ 2\cos x - 2\tan x & 0 & 0 \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\Rightarrow -1(2) + 1(-2) + 1(2) = -2$$

39. If \vec{a}, \vec{b} , and \vec{c} are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$, then $|\vec{a} \times \vec{c}|$ is equal to

(1) $\frac{15}{16}$

(2) $\frac{\sqrt{15}}{16}$

(3*) $\frac{\sqrt{15}}{4}$

(4) $\frac{1}{4}$

Sol. $\vec{a} + 2\vec{c} = -2\vec{b}$

$$|\vec{a} + 2\vec{c}|^2 = |-2\vec{b}|^2$$

$$1 + 4 + 4 \vec{a} \cdot \vec{c} = 4$$

$$\vec{a} \cdot \vec{c} = -\frac{1}{4}$$

$$\cos \theta = -\frac{1}{4}$$

$$\sin \theta = \pm \sqrt{1 - \frac{1}{16}} = \pm \frac{\sqrt{15}}{4}$$

$$\text{Now } |\vec{a} \times \vec{c}| = |\sin \theta| = \left| \pm \frac{\sqrt{15}}{4} \right| = \frac{\sqrt{15}}{4}$$

40. If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $(3 \cos \theta, \sqrt{3} \sin \theta)$ and $(-3 \sin \theta, \sqrt{3} \cos \theta)$, $\theta \in \left(0, \frac{\pi}{2}\right)$. then $\frac{2 \cot \beta}{\sin 2\theta}$ is equal to

- (1*) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{3}}$

Sol. $\frac{dy}{dx} = \frac{-x}{3y}$

Normal $\left(\frac{-1}{dy/dx}\right)_{(3 \cos \theta, \sqrt{3} \sin \theta)} = \sqrt{3} \tan \theta = m_1$

Normal $\left(\frac{-1}{dy/dx}\right)_{(-3 \sin \theta, \sqrt{3} \cos \theta)} = -\sqrt{3} \cot \theta = m_2$

Now $\tan \beta = \left| \frac{\sqrt{3} \tan \theta + \sqrt{3} \cot \theta}{1 - 3} \right| = \frac{\sqrt{3}}{\sin 2\theta}$

Now $\sin 2\theta = \sqrt{3} \cot \beta$

So, $\frac{2 \cot \beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$

41. Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals

- (1) $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$ (2*) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Now $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

$\begin{bmatrix} a & 2a + 3b \\ c & 2c + 3d \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

$c = 0, 2a + 3b = 0$ and $a = 2c + 3d \Rightarrow a = 3d$

$|3A| = 9ad = 108$

$\Rightarrow ad = 12$

$\Rightarrow 3d^2 = 12 \quad [\because a = 3d]$

$d = \pm 2$

So, $a = \pm 6$ [Take both a and d positive or negative because $ad =$ positive]

$A^2 = \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

42. If n is the degree of the polynomial, $\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to
 (1) $(12, 8(10)^4)$ (2) $(24, (10)^8)$ (3) $(8, 5(10)^4)$ (4*) $(12, (20)^4)$

Sol. Here $m = 12$

Let $a = \sqrt{5x^3+1}$ and $b = \sqrt{5x^3-1}$

So, $\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$
 $= 2[{}^8C_0 a^8 + {}^8C_2 a^6 b^2 + {}^8C_4 a^4 b^4 + {}^8C_6 a^2 b^6 + {}^8C_8 b^8]$
 $= 2[(5x^3+1)^4 + 28 \cdot (5x^3+1)^3 (5x^3-1) + {}^8C_4 (5x^3+1)^2 \cdot (5x^3-1)^2$
 $\qquad\qquad\qquad + {}^8C_6 (5x^3+1) (5x^3-1)^3 + (5x^3-1)^4]$

Coefficient of x^{12}
 $= 2[5^4 + 28 \cdot 5^4 + 70 \cdot 5^4 + 28 \cdot 5^4 + 5^4]$
 $= 2^8 \times 5^4 = 4^4 \times 5^4 = (20)^4$

43. The mean of a set of 30 observations is 75. If each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to
 (1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{10}{3}$ (4*) $\frac{4}{3}$

Sol. $\bar{x} = 75$

$\bar{x}_1 = \lambda \bar{x}$

$\bar{x}_2 = 75\lambda$

$= 75\lambda - 25$

ATQ, $75\lambda - 25 = \bar{x} = 75$

$\lambda = \frac{100}{75} \Rightarrow \lambda = \frac{4}{3}$

44. Let S be the set of all real values of k for which the system of linear equations

$x + y + z = 2$
 $2x + y - z = 3$
 $3x + 2y + kz = 4$

has a unique solution. Then S is

- (1) an empty set (2) equal to \mathbb{R} (3*) equal to $\mathbb{R} - \{0\}$ (4) equal to $\{0\}$

Sol. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix}$

$|A| \neq 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$

$k \neq 0$

$R = \{0\}$

45. Consider the following two binary relations on the set $A = \{a, b, c\}$

$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$

and $R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)\}$.

Then

- (1) both R_1 and R_2 are transitive
- (2*) R_2 is symmetric but it is not transitive
- (3) both R_1 and R_2 are not symmetric
- (4) R_1 is not symmetric but it is transitive

Ans. 2

Sol. $(a, c) \in R_1$ but $(c, a) \notin R_1 \Rightarrow R_1$ is not symmetric

$(b, c), (c, a) \in R_1$ but $(b, a) \notin R_1 \Rightarrow R_1$ is not transitive

R_2 is symmetric

and $(c, a), (a, b) \in R_2$ but $(c, b) \notin R_2$

$\Rightarrow R_2$ is not transitive

46. If a right circular cone, having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is

- (1) $8\sqrt{2}\pi$
- (2) $6\sqrt{3}\pi$
- (3) $6\sqrt{2}\pi$
- (4*) $8\sqrt{3}\pi$

Sol. Given $R = 3$

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi [R^2 \cos^2 \theta] [3 + R \sin \theta]$$

$$V = \frac{1}{3} 9\pi \times 3[\cos^2 \theta] [1 + \sin \theta]$$

$$\frac{dV}{d\theta} = 9\pi [-2 \sin \theta \cos \theta (1 + \sin \theta) + \cos^2 \theta (\cos \theta)]$$

$$\frac{dV}{d\theta} = 0 \quad \Rightarrow \quad \sin \theta = \frac{1}{3}$$

$$< 0 \quad \text{at } \sin \theta = \frac{1}{3}$$

So, volume is maximum at $\sin \theta =$

Now curved surface area of cone = $\pi r l$

$$= \pi \times R \cos \theta \times l \quad (\because l = \sqrt{16+8} = \sqrt{24})$$

$$= \pi \times 3 \times \frac{2\sqrt{2}}{3} \times (\sqrt{24})$$

$$= 8\sqrt{3}\pi$$

47. If $f\left(\frac{x-4}{x+2}\right) = 2x + 1$, ($x \in \mathbb{R} - \{1, -2\}$), then $\int f(x) dx$ is equal to

(where C is a constant of integration)

(1) $12 \log_e |1-x| - 3x + C$

(2) $12 \log_e |1-x| + 3x + C$

(3) $-12 \log_e |1-x| + 3x + C$

(4*) $-12 \log_e |1-x| - 3x + C$

Sol. Let $\frac{x-4}{x+2} = t$

$$f(t) = \frac{12}{1-t} - \frac{3(1-t)}{(1-t)}$$

$$f(t) = \frac{12}{1-t} - 3$$

$$\begin{aligned} \text{Now } \int f(x) dx &= \int \frac{12}{1-x} dx - \int 3 dx \\ &= -12 \ln |1-x| - 3x + C \end{aligned}$$

48. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$,

where $f(x) = \begin{cases} 1, & x \in [0, 1] \\ 0, & \text{otherwise} \end{cases}$

If $y(0) = 0$, then $y\left(\frac{3}{2}\right)$ is

(1) $\frac{e^2 - 1}{e^3}$

(2) $\frac{1}{2e}$

(3) $\frac{e^2 + 1}{2e^4}$

(4*) $\frac{e^2 - 1}{2e^3}$

Sol. $\frac{dy}{dx} + 2y = f(x)$

$$\text{I.F.} = e^{\int 2 dx} = e^{2x}$$

$$y \cdot e^{2x} = \int f(x) \cdot e^{2x} dx$$

$$= \begin{cases} \frac{1}{2}e^{2x} + c_1 & \text{if } x \in [0, 1] \\ c_2 & \text{if } x \in \mathbb{R} - [0, 1] \end{cases}$$

$$y = \begin{cases} \frac{1}{2} + c_1 e^{-2x} & \text{if } x \in [0, 1] \\ c_2 e^{-2x} & \text{if } x \in \mathbb{R} - [0, 1] \end{cases}$$

Now $y(0) = 0 \Rightarrow c_1 = 0 \Rightarrow c_1 = V$

$$y = \begin{cases} \frac{1}{2} - \frac{1}{2}e^{-2x} & \text{if } x \in [0, 1] \\ c_2 e^{-2x} & \text{if } x \in \mathbb{R} - [0, 1] \end{cases}$$

Now y is continuous at $x = 1$

$$\Rightarrow \frac{1}{2} - \frac{1}{2}e^{-2} = c_2 e^{-2} = \frac{1}{2}e^2 - \frac{1}{2} = c_2$$

$$\Rightarrow \frac{e^2 - 1}{2} = c_2$$

Now $y = \left(\frac{e^2 - 1}{2}\right)e^{-2x}$ if $x \in \mathbb{R} - [0, 1]$

$$y\left(\frac{3}{2}\right) = \left(\frac{e^2 - 1}{2}\right)e^{-3} = \frac{e^2 - 1}{2e^3}$$

49. The value of the integral $\int_{-\pi/2}^{\pi/2} \sin^4 x \left(1 + \log\left(\frac{2 + \sin x}{2 - \sin x}\right)\right) dx$ is

- (1*) $\frac{3}{8}\pi$ (2) $\frac{3}{16}\pi$ (3) 0 (4) $\frac{3}{4}$

Sol. $\int_{-\pi/2}^{\pi/2} \sin^4 x dx + \int_{-\pi/2}^{\pi/2} \sin^4 x \log\left(\frac{2 + \sin x}{2 - \sin x}\right) dx$
 $2 \int_0^{\pi/2} \left(\frac{1 - \cos 2x}{2}\right)^2 dx + 0$ (\because odd function)

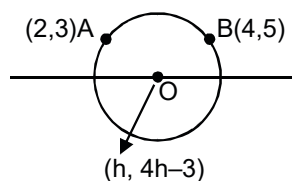
$$2 \int_0^{\pi/2} \frac{1}{4} \left(\frac{3}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x\right) dx = \frac{2}{4} \left[\frac{3}{2}x + \frac{1}{8} \sin 4x - \sin 2x\right]_0^{\pi/2}$$

$$= \frac{1}{2} \left[\frac{3}{2} \times \frac{\pi}{2}\right] = \frac{3\pi}{8}$$

50. A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to

- (1*) 2 (2) 1 (3) $\sqrt{5}$ (4) $\sqrt{2}$

Sol. Now $OA^2 = OB^2$
 $(h - 2)^2 + (4h - 6)^2 = (h - 4)^2 + (4h - 8)^2$
 $\Rightarrow h = 2 \Rightarrow O(2, 5)$
 Now $OA = \sqrt{2^2} = 2$



51. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is

- (1) $x + 2y + 3 = 0$ (2) $8(2x + y) + 3 = 0$ (3) $3(x + y) + 4 = 0$ (4*) $4(x + y) + 3 = 0$

Sol. $x^2 = 4a_1y$ and $y^2 = 4a_2x$ are two parabolas a.t. and

Given $4a_1 = 3$ and $4a_2 = 3$

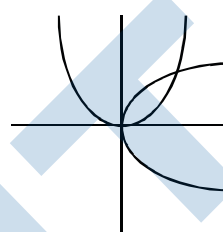
$\Rightarrow x^2 = 3y$ and $y^2 = 3x$

So, common tangent equation

$\Rightarrow xa_1^{1/3} + ya_2^{1/3} + (a_1a_2)^{2/3} = 0$

$\Rightarrow x\left(\frac{3}{4}\right)^{1/3} + y\left(\frac{3}{4}\right)^{1/3} + \left(\frac{3}{4} \cdot \frac{3}{4}\right)^{2/3} = 0$

$\Rightarrow 4(x + y) + 3 = 0$



52. An angle between the plane, $x + y + z = 5$ and the line of intersection of the planes, $3x + 4y + z - 1 = 0$ and $5x + 8y + 2z + 14 = 0$, is

- (1) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (2) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (3*) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$ (4) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Sol. Let D.rs of line of intersection is (a, b, c)

So, $3a + 4b + c = 0$ (1)

and $5a + 8b + 2c = 0$ (2)

by (1) and (2) $a = 0, b = -1$ and $c = 4$

Now $\cos(90^\circ - \theta) = \sin \theta = \frac{|0 \times 1 + 1 \times 1 + 4 \times 1|}{\sqrt{1+16}\sqrt{3}}$

$\sin \theta = \sqrt{\frac{3}{17}}$

53. The set of all $\alpha \in \mathbb{R}$, for which $w = \frac{1 + (1 - 8\alpha)z}{1 - z}$ is a purely imaginary number, for all $z \in \mathbb{C}$ satisfying

$|z| = 1$ and $\text{Re } z \neq 1$, is

- (1) equal to \mathbb{R} (2*) $\{0\}$ (3) $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$ (4) an empty set

Sol. Let $z = x + iy$

$$w = \frac{1 + (1 - 8\alpha)(x + iy)}{(1 - x) + iy}$$

$$w = \frac{[1 + (1 - 8\alpha)(x + iy)][(1 - x) - iy]}{(1 - x)^2 + y^2}$$

w is purely imaginary so, $R_e(z) = 0$

$$8\alpha(-x + 1) = 0 \quad [\because x^2 + y^2 = 1]$$

Given $x \neq 1$ So, $\alpha = 0$

54. If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point $(-2, 0)$ is

- (1*) -34 (2) -2 (3) -32 (4) 4

Sol. $2x + 2y y' + \cos y y' = 0$

$$y' = \frac{d^2y}{dx^2} \Rightarrow y'_{(-2,0)} = 4$$

$$y'' = \frac{-2(2y + \cos y) - (-2x)(2yy' - \sin yy')}{(2y + \cos y)^2}$$

$$y''_{(-2,0)} = -34$$

55. If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3 \sin^2(A + B) - 10 \sin(A + B) \cdot \cos(A + B) - 25 \cos^2(A + B)$ is

- (1) 25 (2*) -25 (3) -10 (4) 10

Sol. $\tan A + \tan B = \frac{10}{3}$ and $\tan A \cdot \tan B = \frac{-25}{3}$

$$\text{So, } \tan(A + B) = \frac{10/3}{1 + 25/3} = \frac{10}{28} = \frac{5}{14}$$

$$\text{So, } \sin(A + B) = \frac{5}{\sqrt{221}} \text{ and } \cos(A + B) = \frac{14}{\sqrt{221}}$$

$$3 \sin^2(A + B) - 10 \sin(A + B) \cdot \cos(A + B) - 25 \cos^2(A + B)$$

$$\frac{3 \times 25}{221} - 10 \times \frac{5}{\sqrt{221}} \times \frac{14}{\sqrt{221}} - \frac{25 \times 196}{221} = \frac{75 - 700 - 4900}{221} = -25$$

56. If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is

- (1) $4x^2 - y^2 - 16x^2y^2 = 0$ (2*) $4x^2 - y^2 + 16x^2y^2 = 0$
 (3) $x^2 - 4y^2 - 16x^2y^2 = 0$ (4) $x^2 - 4y^2 + 16x^2y^2 = 0$

Sol. $x^2 - 4y^2 = -1$

Equation of tangent $y = mx \pm \sqrt{a^2m^2 - b^2}$

$$y = mx \pm \sqrt{m^2 - \frac{1}{4}}$$

57. In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are respectively, $x + y = 5$ and $x = 4$. Then area of ΔABC (in square units) is

- (1) 4 (2*) 9 (3) 5 (4) 12

Sol. Intersection point of BE and CF is (4, 1) (Centroid of ΔABC)

Let C(4, α), then point B will be (7, r)

And $E = \left(\frac{5}{2}, \frac{2+\alpha}{2}\right)$ lie on line $x + y = 5$

So, $\frac{5}{2} + \frac{2+\alpha}{2} = 5$

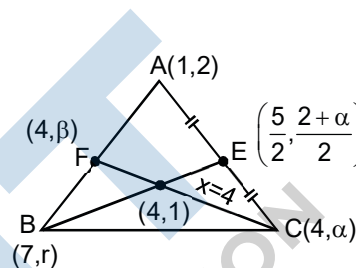
$\alpha = 3 \Rightarrow C(4, 3)$

$\beta \equiv (7, r)$ lie on line $x + y = 5$

So, $7 + r = 5 \Rightarrow r = -2$

$\beta \equiv (7, -2)$

area of $\Delta = \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 7 & -2 & 1 \\ 4 & 3 & 1 \end{vmatrix} = 9$ square unit



58. Let $S = \{(\lambda, \mu) \in \mathbb{R} \times \mathbb{R} : f(t) = (|\lambda| e^{|\mu|} - \mu) \cdot \sin(2|t|), t \in \mathbb{R}, \text{ is a differentiable function}\}$. Then S is a subset of

- (1) $[0, \infty) \times \mathbb{R}$ (2) $(-\infty, 0) \times \mathbb{R}$ (3) $\mathbb{R} \times (-\infty, 0)$ (4*) $\mathbb{R} \times [0, \infty)$

Sol. $f(t) = (|\lambda| e^{|\mu|} - \mu) \cdot \sin(2|t|)$

is not differentiable at $t = 0$

So, $f(t)$ is differentiable only if $(|\lambda| e^{|\mu|} - \mu) = 0$

at $t = 0$

$|\lambda| = \mu \Rightarrow \mu = [0, \infty)$

So, S is subset of $\mathbb{R} \times [0, \infty)$

59. A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is

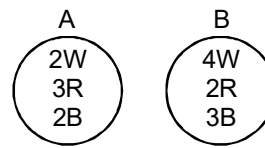
(1) $\frac{9}{32}$

(2) $\frac{9}{16}$

(3) $\frac{7}{8}$

(4*) $\frac{7}{16}$

- Sol.** A → Balls are drawn from bag A
 B → Balls are drawn from bag B
 W → Ball is White
 R → Ball is Red
 B → Ball is Black



$$P\left(\frac{B}{RW}\right) = \frac{P(B) P\left(\frac{RW}{B}\right)}{P(A) P\left(\frac{RW}{A}\right) + P(B) P\left(\frac{RW}{B}\right)} = \frac{\frac{1}{2} \times \frac{2 \times 4}{{}^9C_2}}{\frac{1}{2} \times \frac{2 \times 4}{{}^9C_2} + \frac{1}{2} \times \frac{2 \times 3}{{}^7C_2}} = \frac{7}{16}$$

- 60.** The area (in sq units) of the region $\{x \in \mathbb{R} : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\}$, is

(1*) $\frac{10}{3}$

(2) $\frac{8}{3}$

(3) $\frac{5}{3}$

(4) $\frac{13}{3}$

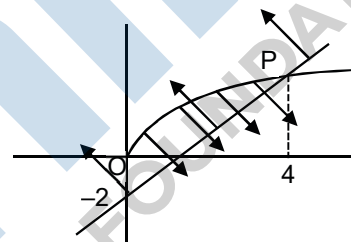
- Sol.** For P, $\sqrt{x} = x - 2$

$$t^2 - t - 2 = 0$$

$$(t - 2)(t + 1) = 0$$

$$t = 2, \quad t = -1$$

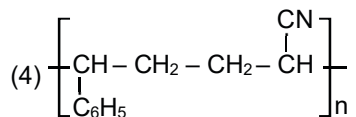
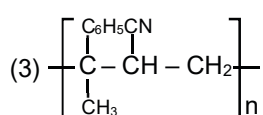
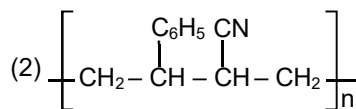
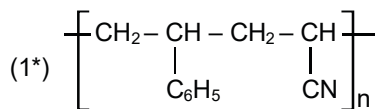
$$x = 4$$



$$\begin{aligned} \text{Required area} &= \int_0^4 \sqrt{x} \, dx - \int_2^4 (x - 2) \, dx \\ &= \frac{2}{3} [x^{3/2}]_0^4 - \left(\frac{x^2}{2} - 2x\right)_2^4 \\ &= \frac{16}{3} - [8 - 8 - (2 - 4)] = \frac{10}{3} \end{aligned}$$

PART-C-CHEMISTRY

61. The copolymer formed by addition polymerization of styrene and acrylonitrile in the presence of peroxide is :



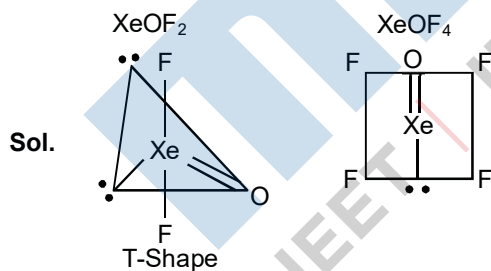
62. The correct match between items of List I and List II is :

List I	List II
(A) Coloured impurity	(P) Steam distillation
(B) Mixture of o-nitrophenol and p-nitrophenol	(Q) Fractional distillation
(C) Crude Naphtha	(R) Charcoal treatment
(D) Mixture of glycerol and sugars	(S) Distillation under reduced pressure

(1) (A) - (R), (B) - (P), (C) - (S), (D) - (Q)
 (2*) (A) - (R), (B) - (P), (C) - (Q), (D) - (S)
 (3) (A) - (R), (B) - (S), (C) - (P), (D) - (Q)
 (4) (A) - (P), (B) - (S), (C) - (R), (D) - (Q)

63. Identify the pair in which the geometry of the species is T-shape and square pyramidal, respectively :

- (1) IO_3^- and IO_2F_2^- (2) ClF_3 and IO_4^-
 (3*) XeOF_2 and XeOF_4 (4) ICl_2^- and ICl_5



64. For Na^+ , Mg^{2+} , F^- and O^{2-} ; the correct order of increasing ionic radii is :

- (1) $\text{Mg}^{2+} < \text{O}^{2-} < \text{Na}^+ < \text{F}^-$ (2*) $\text{Mg}^{2+} < \text{Na}^+ < \text{F}^- < \text{O}^{2-}$
 (3) $\text{Na}^+ < \text{Mg}^{2+} < \text{F}^- < \text{O}^{2-}$ (4) $\text{O}^{2-} < \text{F}^- < \text{Na}^+ < \text{Mg}^{2+}$

Sol. Isoelectronic series : $\text{Mg}^{2+} < \text{Na}^+ < \text{F}^- < \text{O}^{2-}$

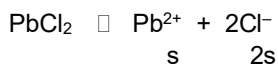
When negative charge increase, increase the radius of ion.

65. The minimum volume of water required to dissolve 0.1 g lead (II) chloride to get a saturated solution.

[K_{sp} of $PbCl_2 = 3.2 \times 10^{-8}$; atomic mass of Pb = 207 u] is :

- (1) 1.798 L (2) 0.36 L (3) 17.98 L (4*) 0.18 L

Sol. (K_{sp})_{PbCl₂} = 32×10^{-9}



$$K_{sp} = [Pb^{2+}][Cl^-]^2$$

$$K_{sp} = 4s^3 = 32 \times 10^{-9}$$

$$s^3 = 8 \times 10^{-9}$$

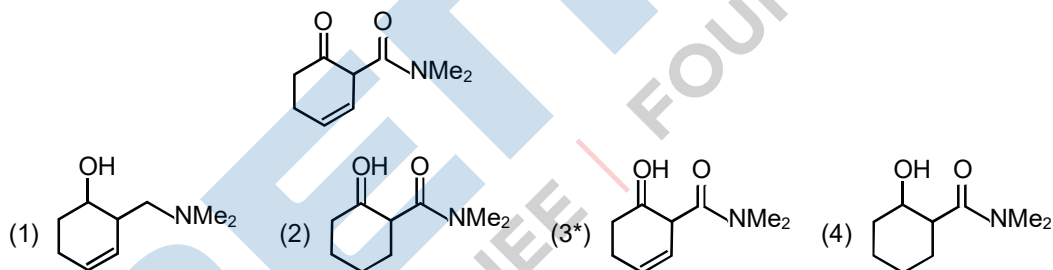
$$s = 2 \times 10^{-3} \text{ M}$$

$$\frac{w}{M.w.} \times \frac{1}{V_L} = 2 \times 10^{-3}$$

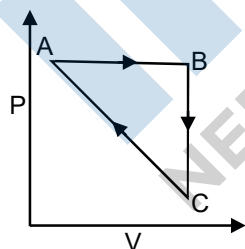
$$\frac{0.1}{278} \times \frac{1}{V_L} = 2 \times 10^{-3}$$

$$V_L = \frac{0.1 \times 1000}{278 \times 2} = 0.18 \text{ L}$$

66. The main reduction product of the following compound with $NaBH_4$ in methanol is :



67. An ideal gas undergoes a cyclic process as shown in figure.



$$\Delta U_{BC} = -5 \text{ kJ mol}^{-1}, q_{AB} = 2 \text{ kJ mol}^{-1}$$

$$W_{AB} = -5 \text{ kJ mol}^{-1}, W_{CA} = 3 \text{ kJ mol}^{-1}$$

Heat absorbed by the system during process CA is :

- (1) 18 kJ mol⁻¹ (2) - 18 kJ mol⁻¹ (3*) + 5 kJ mol⁻¹ (4) - 5 kJ mol⁻¹

- Sol.** AB → isobaric
 BC → isochoric
 CA → not defined

$$\Delta U_{AB} = q + W$$

$$= 2 - 5 = -3$$

$$\Delta U_{ABC} = \Delta U_{AB} + \Delta U_{BC}$$

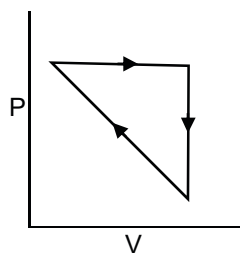
$$= -3 - 5 = -8 \text{ kJ}$$

$$\Delta U_{CBA} = +8$$

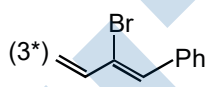
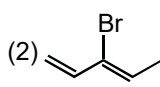
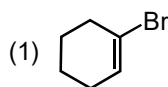
$$= Q + W$$

$$8 = Q + 3$$

$$Q = +5 \text{ kJ}$$

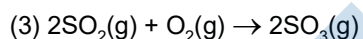
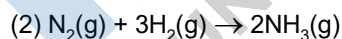
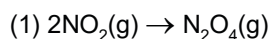


68. Which of the following will most readily give the dehydrohalogenation product?



- Sol.** Here dehydrohalogenation goes by E1cb and most stable carbanion formation is favoured in A.

69. For which of the following reactions, ΔH is equal to ΔU ?



- Sol.** $\Delta H = \Delta U + \Delta n_g RT$



70. Ejection of the photoelectron from metal in the photoelectric effect experiment can be stopped by applying 0.5 V when the radiation of 250 nm is used. The work function of the metal is :

(1) 4 eV

(2) 5.5 eV

(3*) 4.5 eV

(4) 5 eV

- Sol.** $\lambda = 250 \text{ nm} = 2500 \text{ \AA}$

$$E = \frac{hc}{\lambda} = \frac{12400}{2500} = 4.96 \text{ eV}$$

$$\text{KE} = \text{stopping potential} = 0.5 \text{ eV}$$

$$E = W_0 + \text{K.E.}$$

$$4.96 = W + 0.5$$

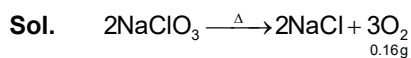
$$W_0 = 4.46 \approx 4.5 \text{ eV}$$

71. The increasing order of nitration of the following compounds is :

74. A sample of NaClO_3 is converted by heat to NaCl with a loss of 0.16 g of oxygen. The residue is dissolved in water and precipitated as AgCl . The mass of AgCl (in g) obtained will be :

[Given : Molar mass of $\text{AgCl} = 143.5 \text{ g mol}^{-1}$]

- (1) 0.41 (2*) 0.48 (3) 0.35 (4) 0.54



$$\frac{n_{\text{NaCl}}}{2} = \frac{n_{\text{O}_2}}{3}$$

$$n_{\text{NaCl}} = \frac{0.16}{32} \times \frac{2}{3} = \frac{1}{200} \times \frac{2}{3} = \frac{1}{300}$$



POAC of Cl

$$1 \times n_{\text{NaCl}} = 1 \times n_{\text{AgCl}}$$

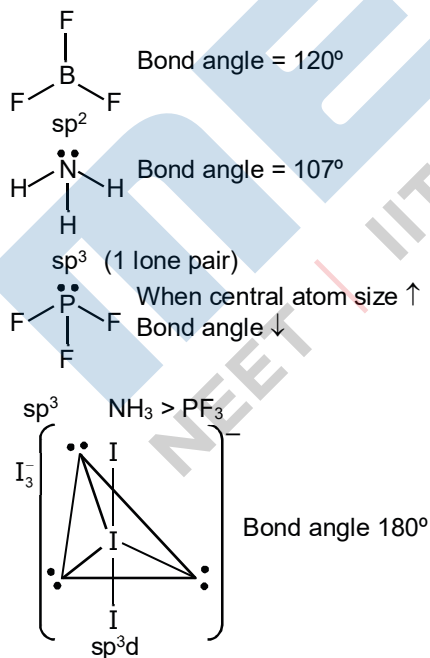
$$\frac{1}{300} = n_{\text{AgCl}}$$

$$\begin{aligned} \text{Weight of AgCl} &= \frac{1}{300} \times [108 + 35.5] = \frac{1}{300} \times 143.5 \\ &= 0.48 \text{ g} \end{aligned}$$

75. The decreasing order of bond angles in BF_3 , NH_3 , PF_3 and I_3^- is :

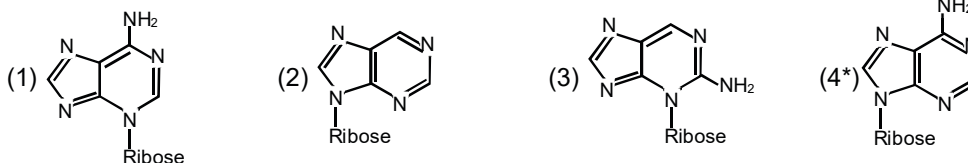
- (1) $\text{BF}_3 > \text{NH}_3 > \text{PF}_3 > \text{I}_3^-$ (2) $\text{BF}_3 > \text{I}_3^- > \text{PF}_3 > \text{NH}_3$
 (3*) $\text{I}_3^- > \text{BF}_3 > \text{NH}_3 > \text{PF}_3$ (4) $\text{I}_3^- > \text{NH}_3 > \text{PF}_3 > \text{BF}_3$

Sol.

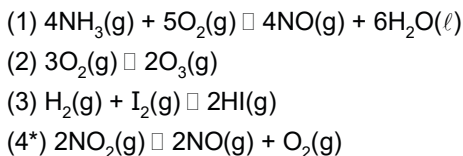


Bond Angle $\text{PF}_3 < \text{NH}_3 < \text{BF}_3 < \text{I}_3^-$

76. Which of the following is the correct structure of Adenosine?



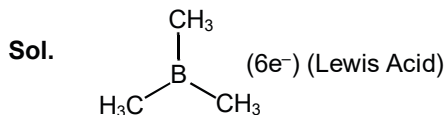
77. In which of the following reactions, an increase in the volume of the container will favour the formation of products?



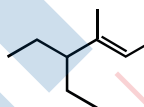
Sol. Volume \uparrow P \downarrow reaction proceed in which direction where number of gases mole increases.



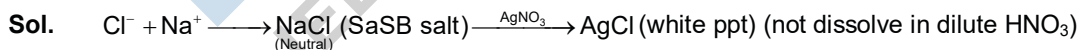
78. Which of the following is a Lewis acid?



79. The IUPAC name of the following compound is :



80. A white sodium salt dissolves readily in water to give a solution which is neutral to litmus. When silver nitrate solution is added to the aforementioned solution, a white precipitate is obtained which does not dissolve in dil. nitric acid. The anion is :



81. Which of the following will not exist in zwitter ionic form at pH = 7 ?

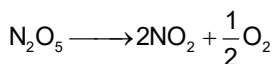


$\% P = \frac{2}{3} \times 100 = 67\%$	$\% P = \frac{3}{4} \times 100 = 75\%$
--	--

86. N_2O_5 decomposes to NO_2 and O_2 and follows first order kinetic. After 50 minutes, the pressure inside the vessel increases from 50 mmHg to 87.5 mmHg. The pressure of the gaseous mixture after 100 minute at constant temperature will be :

- (1) 116.25 mmHg (2) 175.0 mmHg (3) 136.25 mmHg (4*) 106.25 mmHg

Sol.



t = 0 50 0 0

t = 50 min. 50 - p₁ 2p₁ $\frac{p_1}{2}$

t = 100 min. 50 - p₂ 2p₂ $\frac{p_2}{2}$

= 12.5

$$50 - p_1 + 2p_1 + \frac{p_1}{2} = 87.5$$

$$50 + \frac{3p_1}{2} = 87.5$$

$$\frac{3p_1}{2} = 37.5$$

$$p_1 = \frac{37.5 \times 2}{3} = 25$$

50 minute is half life period

for 100 minute (2 half life)

$$50 - p_2 = 12.5$$

$$p_2 = 37.5 \text{ mm of Hg}$$

Total pressure at 100 minute

$$= 50 - p_2 + 2p_2 + \frac{p_2}{2}$$

$$= 50 + \frac{3p_2}{2} = 50 + \frac{3}{2} \times 37.5$$

$$= 50 + 56.25$$

$$= 106.25 \text{ mm of Hg}$$

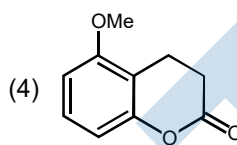
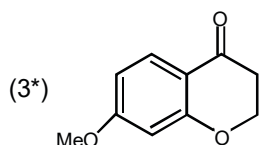
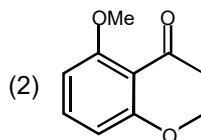
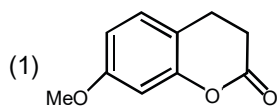
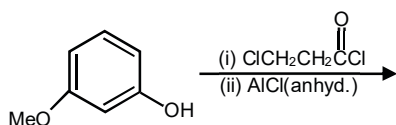
87. Which of the following statements about colloids is False?

- (1) When silver nitrate solution is added to potassium iodide solution, a negatively charged colloidal solution is formed.
- (2) When excess of electrolyte is added to colloidal solution, colloidal particle will be precipitated.
- (3) Colloidal particles can pass through ordinary filter paper.

(4*) Freezing point of colloidal solution is lower than true solution at same concentration of a solute.

Sol. Freezing point of colloidal solution is higher than true solution at same concentration of a solute.

88. The major product of the following reaction is :



Sol. The reactant undergoes acylation first followed by substitution Intramolecular.

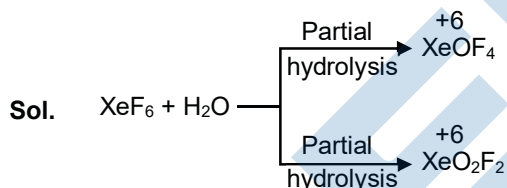
89. Xenon hexafluoride on partial hydrolysis produces compounds X and Y . Compounds X and Y and the oxidation state of Xe are respectively :

(1) XeO_2F_2 (+ 6) and XeO_2 (+ 4)

(2) XeOF_4 (+ 6) and XeO_3 (+ 6)

(3) XeO_2 (+ 4) and XeO_3 (+ 6)

(4*) XeOF_4 (+ 6) and XeO_2F_2 (+ 6)



90. Which of the following arrangements shows the schematic alignment of magnetic moments of antiferromagnetic substance?



Sol. Substances which are expected to possess para-magnetism or ferro-magnetism on the basis of unpaired electrons but actually they possess zero net magnetic moment are called anti ferromagnetic substance.